THREE-DIMENSIONAL VORTEX STRUCTURES IN CAVITIES

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There are two main approaches in theoretical studies on the flow of liquids in planar cavities.

The first treats the liquid as ideal. Prandtl was the first to suggest constancy of the vorticity with closed current lines. This assumption was subsequently demonstrated rigorously [1] for a two-dimensional region. It is necessary to link up the external potential flow above the cavity and the flow of constant vorticity in the cavity [2]. A solution to this problem has been obtained [3].

The second approach incorporates the effects of viscosity in the mixing zone (above the upper edge of the slot) and in the boundary layers (along all the walls of the cavity). In that case, the vorticity is determined by considering the balance of the forces at the mobile and immobile boundaries of the vortex [4-6]. Numerical solutions to the Navier—Stokes equations have also been obtained, but only in the two-dimensional case. Numerous experiments show that there are three-dimensional flows in planar cavities behind steps and before ridges. Measurements have been made [7, 8] of the pressure distribution at the bottom and the forward wall of the cavity, with flow vizualization at the bottom with an oil—carbon black mixture, which revealed three-dimensional structures for shallow cavities $H/L > 0.4$ in a turbulent flow. Here H and L are the depth and chord of the cavity. Stroboscopic measurement of the velocity profile behind a step has been used [9], as has laser Doppler measurement [10] of the velocity, which indicated that there are three-dimensional flows in the laminar and turbulent states. The structure of the three-dimensional formations remains unclear. Here we examine the vortex flows in two-dimensional cavities.

The experiments were performed on an apparatus with a planar channel of section 10×100 mm ($h = 10$, $B = 100$ mm); the motion of the working liquid (distilled water) in the channel was provided by the difference in heights of the upper and lower constant-level tanks. The water from the upper tank passed through a settling chamber (cross section 120×100 mm) with a honeycomb and confuser to the working channel, whose length was 2000 mm. The honeycomb was made of thin-walled nickel tubes (wall thickness 0.1 mm, diameter 5 mm, length 50 mm). The cavity was formed by the sudden expansion of one of the walls of the working channel at a distance of 1500 mm from the inlet. The water flow rate was measured with a diaphragm placed after the working part before the outlet to the lower tank. The water temperature was kept constant.

The flow was vizualized by the addition of aluminum powder of mean particle size 30 μm. A flat light guide (thickness ~ 2 mm) was used to examine and photograph the flow in different planes parallel to the bottom of the cavity. We used inserts that varied the depth of the cavity while maintaining the chord unchanged at $L = 40$ mm (the chord L is the size of the cavity in the flow direction). Figure 1 shows the system.

It was found that at a Reynolds number $Re = 500$ (as calculated from the mean flow-rate velocity in the channel and the transverse dimension h of the channel) there were secondary paired vortex formations in

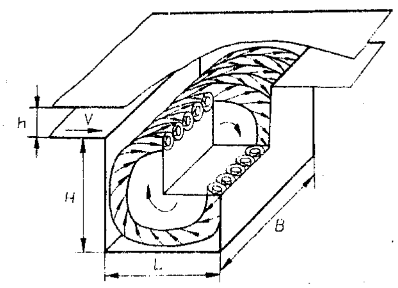

Fig. 1

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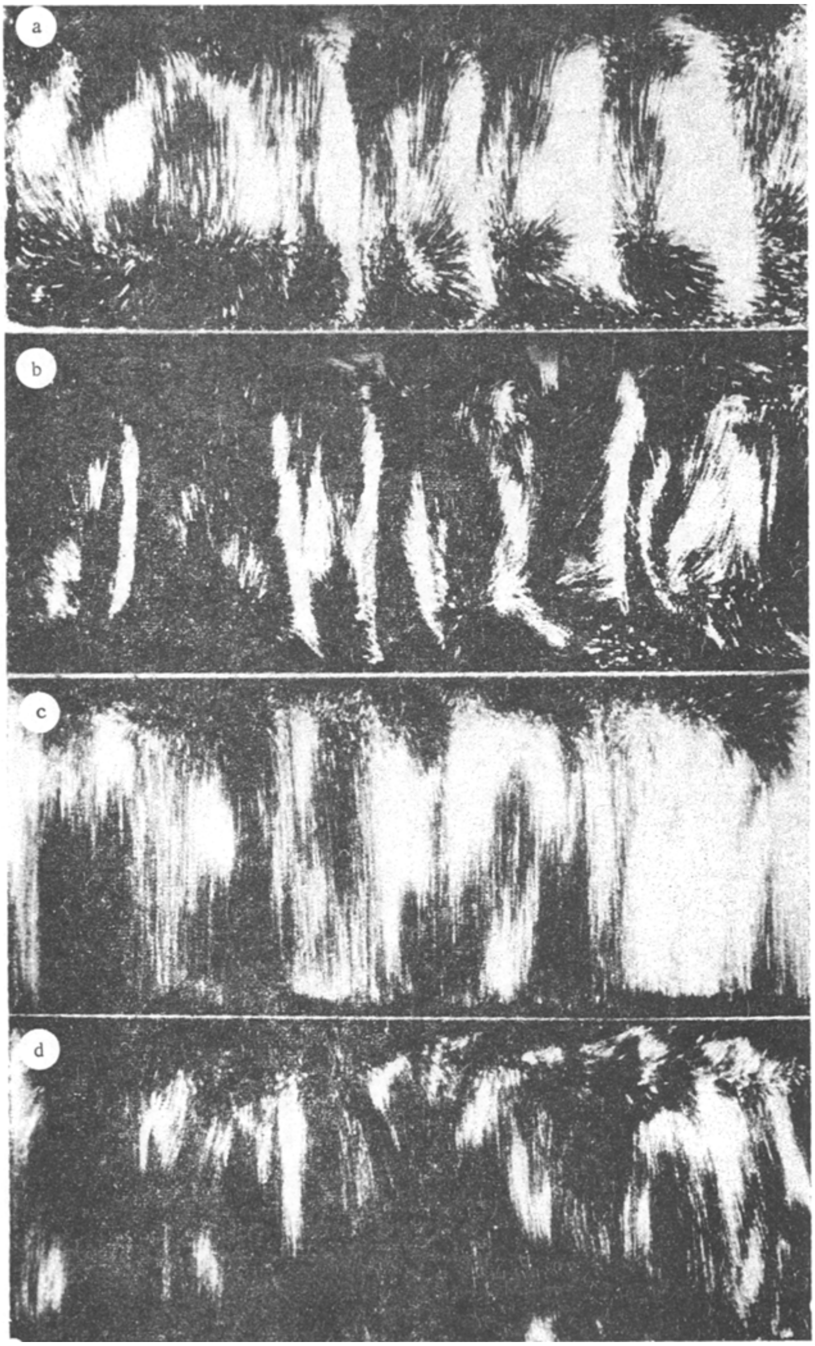

Fig. 2

the cavity between the core of the flow and the walls. Parts *a* and b of Fig. 2 show photographs of these vortices in a plane parallel to the bottom of a square cavity (2 mm from the bottom) for Re = 1000 and 3000. There were these vortex circulations on all the walls of the cavity and in the mixing zone. The flow picture near the mixing region is shown in parts c and d of Fig. 2 for a plane parallel to the bottom of the cavity and separated from it by a distance of 38 mm. The structure of the flow in a plane passing through the axis of the main vortex (20 mm from the bottom) is shown in Fig. 3 for Re = 1000. The upper part of the picture shows secondary paired vortices at the rear wall. The structures are Taylor–Goertler vortices arising in the wall because of curvature of the flow lines and the effects of centrifugal forces. These vortex loops move slowly across the width of the cavity and alter somewhat in outline. The photographs also show the three-dimensional character of the flow at the center of the cavity.

Measurements have been made [10] of the velocity profiles in a cross section of the cavity of the same configuration for Re = 500. The data allow one to estimate the Goertler number $N = (U\delta/\nu)\sqrt{\delta/R} \simeq 20$.

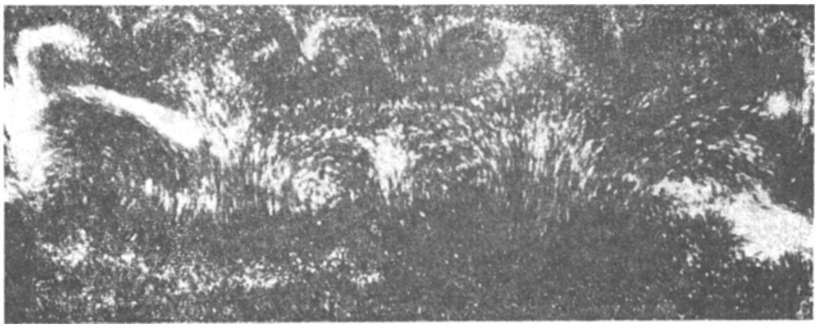

Fig. 3

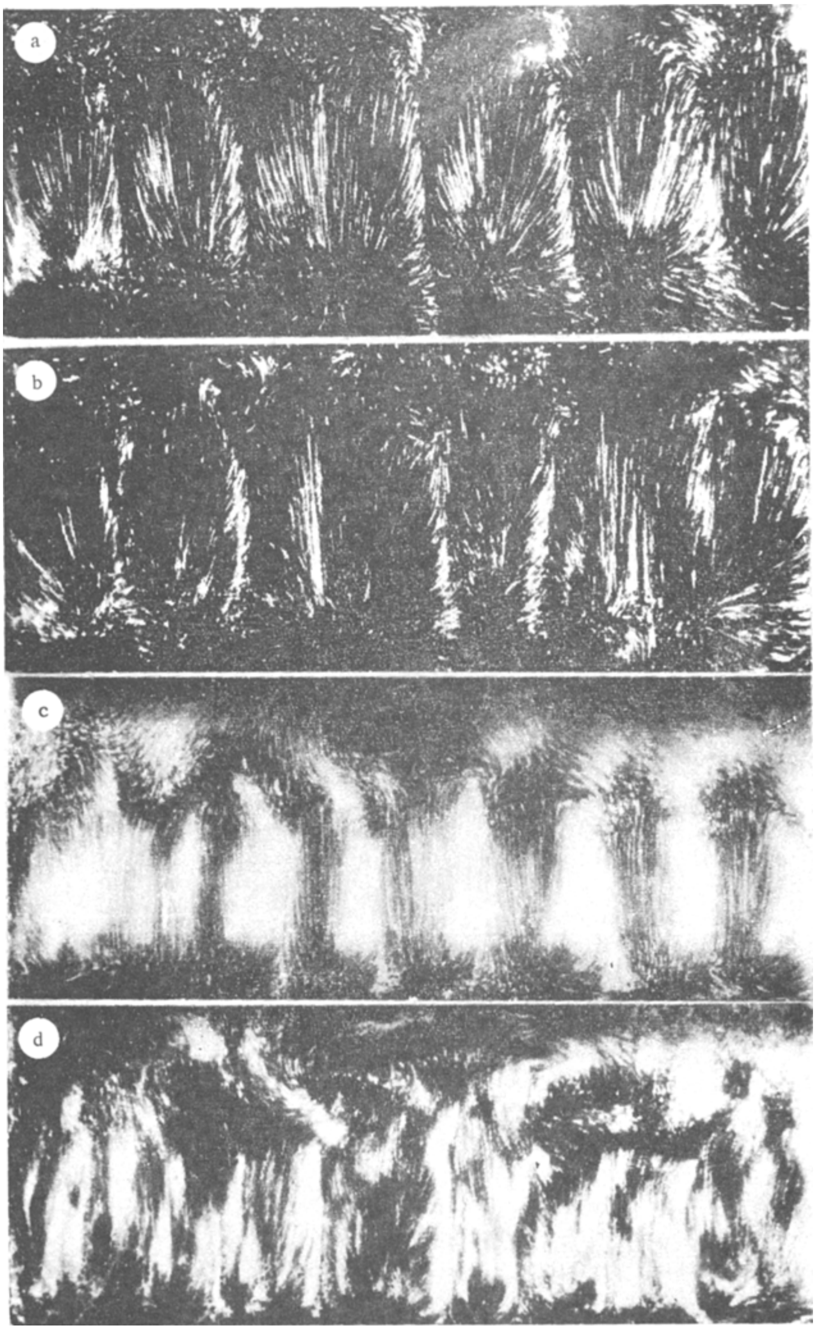

Fig. 4

where $U \simeq 4 \cdot 10^{-3}$ m/sec is the velocity at the boundary of the boundary layer, $\delta \simeq 8 \cdot 10^{-3}$ m is the boundary-layer thickness, $R \simeq H/2 = 20 \cdot 10^{-3}$ m is the radius of the cavity, and ν is the viscosity of water.

The critical value of the Goertler number for concave plates is $N_* = 16$ [11], and this shows that the flow in the cavity has the conditions necessary for the production of Taylor–Goertler structures.

Parts *a* and b of Fig. 4 show the flow pattern at the bottom of a rectangular cavity with $H/L = 0.875$ for Re of 1000 and 3000, while parts c and d of Fig. 4 are for a shallow cavity with $H/L = 0.525$ (and Re of 1000 and 3000, respectively). In the case of the shallow cavity, the main vortex takes up only part of the cross section and adjoins the rear wall. Secondary vortices are formed only near the main vortex. As Re increases from 500 to 3000, the distance λ between adjacent pairs of rings decreases. Also, λ increases with the depth of the cavity (for given Re and a given chord). For example, $\lambda = 16$ mm for $H/L = 0.875$ for $Re = 1000$, while $\lambda = 10$ mm for $H/L = 0.525$ with the same value of Re.

In the range of Re used, we made experiments on a square cavity with $H/L = 1$ of less width with the same width for the channel (widths of the cavity and channel 70 and 100 mm, respectively). For this purpose transparent inserts were placed at the ends of the cavity. These experiments showed that the total number of vortices was reduced in accordance with the reduced width of the cavity, while the distance between adjacent pairs of vortices remained as in a cavity of width 100 mm.

Flow vizualization for the channel showed that the flow was of mobile character for $Re \simeq 3500$. When the turbulent formations (turbulent spots) pass through the part of the channel with the cavity, there was an increase in the flow velocity in the latter and break-up in the cellular structure. After the passage of a turbulent plug, the original velocity was restored in the cavity together with the cellular structure. No cellular structure was observed in the cavity for a flow with $Re = 5000$.

This experimental study therefore shows that a planar laminar stabilized flow uniform over the width in the channel before the cavity (with Re of 500–4500 in our conditions) gives a flow in the cavity of three-dimensional character. There are secondary vortex structures of Taylor–Goertler type in the wall zone, which are analogous to those occurring in flow around concave surfaces.

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